Section 1:
(Please answer two questions from those below)

1. Prove that the Dutch Book argument is consistent with the following axiom of probability:
   \[ 0 \leq \Pr(E) \leq 1 \]
   (All probabilities must be between 0 and 1)
   (note: you must prove this both forwards and backwards)

2. Prove that the Dutch Book argument is consistent with the following axiom of probability:
   \[ \Pr(\Omega) = 1 \]
   ( Necessary events have a probability of 1; Tautologies have a probability of 1 )
   (note: you must prove this both forwards and backwards)

Problems 1 & 2 give us two of the three axioms of probability (or versions of them, at least). The third axiom of probability is the Additivity axiom:
   \[ \Pr(E \cup F) = \Pr(E) + \Pr(F) \]

3. Prove axiomatically that:
   (i) \( \Pr(p) = 1 - \Pr(\neg p) \); and
   (ii) if \( x \) and \( y \) mutually entail one another, then \( \Pr(x) = \Pr(y) \)

4. Marilyn vos Savant is in the Guinness Book of Records as having the highest recorded IQ ever measured, and has a weekly column in Parade magazine called, “Ask Marilyn”. A reader once wrote her with the following question:

   Suppose you’re on a game show, and you’re given the choice of three doors. Behind one door is a car, the others, goats. You pick a door, say #1, and the host, who knows what’s behind the doors, opens another door, say #3, which has a goat. He says to you, "Do you want to pick door #2?" Is it to your advantage to switch your choice of doors?

Marilyn’s answer was that it is to your advantage to switch doors. This has come to be known as the Monty Hall Problem (given the game show the problem most resembles). Do you agree with Marilyn? Explain your reasoning, and support your position using one of the formal methods learned in class.
Section 2:
(Please answer two questions from those below)

5. Consider the following passage:

All this means that the frequency theorist is forced to introduce a modification of his theory—apparently a very slight one. He will now say that an admissible sequence of events (a reference sequence, a ‘collective’) must always be a sequence of repeated experiments. Or more generally, he will say that admissible sequences must be either virtual or actual sequences which are characterised by a set of of generating conditions—by a set of conditions whose repeated realisation produces the elements of the sequence.

...Moreover, it seems that what I have here described as a ‘modification’ only states explicitly an assumption which most frequency theorists (myself included) have always taken for granted.

Yet, if we look more closely at this apparently slight modification, then we find that it amounts to a transition from the frequency interpretation to the propensity interpretation. The frequency interpretation always takes probability as relative to a sequence which is assumed as given; and it works on the assumption that a probability is a property of some given sequence. But with our modification, the sequence in its turn is defined by its set of generating conditions; and in such a way that probability may now be said to be a property of the generating conditions.


Explain the significance of the ‘slight modification’ introduced by Popper. What does Popper mean that probability is best interpreted as a propensity?

6. Consider the following passage:

Imagine that I draw a ‘straight line’ on a blackboard with a piece of chalk. What a complicated thing is this ‘line’ compared with the ‘straight line’ defined by geometry! In the first place, it is not a line at all, since it has definite breadth; even more than that, it is a three-dimensional body made of chalk, an aggregate of many small bodies, the chalk particles. ... All the same, we do know that the exact idealized conceptions of pure geometry are essential tools for dealing with the things around us. We need these abstract concepts just because they are simple enough that our minds can handle them with comparative ease.

...I am prepared to concede without further argument that all the theoretical constructions, including geometry, which are used in the various branches of physics are only imperfect instruments to enable the world of empirical facts to be reconstructed in our minds. The theory of probability, which we include among the exact sciences, is just one such theoretical system.


Discuss how this passage is relevant to von Mises’ theory of probability. In particular, discuss how this relates to his slogan, “First the Collective, Then the Probability.”
7. LaPlace describes probability as being relative in part to our ignorance, and in part to our knowledge of the world. Other probability theoreticians have also noticed the multi-faceted nature of probability. Hacking, for example:

... probability ... is Janus-faced. On the one side it is statistical, concerning itself with stochastic laws of chance processes. On the other side it is epistemological, dedicated to assessing reasonable degrees of belief in propositions quite devoid of statistical background.

Hacking I (1975) *The Emergence of Probability.*

Here Hacking describes the two ‘sides’ of probability as being statistical and epistemological. Other descriptions have included objective and subjective. These imply a pluralist stance towards probability. How do you think this Janus-faced nature of probability should be accounted for? Propose terms to describe the various ‘sides’ (you may use those that have already been proposed, as long as you explain why you think they are good options). If you take a monist stance, then account for this Janus-faced nature described by Hacking and others.

8. Two of the most influential and important contemporary Bayesians are Colin Howson and Peter Urbach. They argue that Bayesian notions of confirmation are both prescriptive and descriptive of scientific reasoning. Consider the following passage from their book, *Scientific Reasoning: The Bayesian Approach*:

Information gathered in the course of observation is often considered to have a bearing on the acceptability of a theory or hypothesis (we use the terms interchangeably), either by confirming it or disconfirming it. Such information may either derive from casual observation or, more commonly, from experiments deliberately contrived in the hope of obtaining relevant evidence. The idea that evidence may count for or against a theory, or be neutral towards it, is a central feature of scientific inference, and the Bayesian account will clearly need to start with a suitable interpretation of these concepts.

Fortunately, there is a suitable and very natural interpretation, for if $P(h)$ measures your belief in a hypothesis when you do not know the evidence $e$, and $P(h \mid e)$ is the corresponding measure when you do, $e$ surely confirms $h$ when the latter exceeds the former. So we shall take the following as our definitions:

- $e$ confirms or supports $h$ when $P(h \mid e) > P(h)$
- $e$ disconfirms or undermines $h$ when $P(h \mid e) < P(h)$
- $e$ is neutral with respect to $h$ when $P(h \mid e) = P(h)$

... Baye’s Theorem relates the posterior probability of a hypothesis, $P(h \mid e)$, to the terms $P(h)$, $P(e \mid h)$, and $P(e)$. Hence, knowing the values of these last three terms, it is possible to determine whether $e$ confirms $h$, and, more importantly, to calculate $P(h \mid e)$. In practice, of course, the various probabilities may only be known rather imprecisely; we shall have more to say about this practical aspect of the question later.
Howson and Urbach are considered to be working on probability in the tradition of Ramsey and De Finetti. Explain why we should consider Howson and Urbach to be subjectivists about probability. Do you think this is a good way to think about scientific reasoning?