Things you need to know for PHIL 3210

Some things about algebra:

When x equals y, any algebraic function, $f$, which is performed on x yielding some value, will yield the same value when performed on y. So:

Applying the function, $f(w) = 1/w$ to both sides of the equality,

\[ a = b \] yields \[ 1 = b/a \]

Let’s work through how that happens.

$f(w) = 1/w$ means that for any value $w$, applying the function described produces $1/w$. So, in the equality above, applying $f(w) = 1/w$ to both sides yields:

\[ 1/a = 1/b \]

Multiplying both sides by $b$ then yields:

\[ b/a = 1 \]

In this class, I assume a basic skill level in algebra. In addition to applying functions to values, I expect students to be able to cancel out and multiply through equations containing functions, variables and other algebraic values. Some of these skills are nicely displayed when we consider how to manipulate fractions:

Canceling out variables:

\[ ab/a = a \]

ex: \[ [(3)(9)]/3 = 9 \]

\[ [(a + b)c]/(a + b) = c \]

ex: \[ [(3 + 6)(3)]/(3 + 6) = 3 \]

You should also know when you may *not* to cancel out:

\[ (a + b)/b \neq a \]

ex: \[ (6 + 3)/3 \neq 6 \]

Multiplying fractions:

\[ (a/b) \times (c/d) = ac/bd \]

ex: \[ (4/2) \times (5/3) = 20/6 = 10/3 \] (notice the simplification)

Dividing fractions:

\[ (a/b)/(c/d) = ad/bc \]

ex: \[ (4/2)/(5/6) = 24/10 = 12/5 \]

(Dividing fractions sometimes causes students grief. Just flip one and multiply.)

Adding fractions:

\[ a/b + c/d = (ad + cb)/bd \]

ex: \[ (4/2) + (5/6) = [(4)(6) + (5)(2)]/12 = 17/6 \]
Some things about summation notation:

Summation notion can be thought of as a function of another function, $g(f(x))$.

$$\sum_{i=m}^{n} f(i) = f(m) + f(m+1) + \ldots + f(n), \text{ where } m \text{ and } n \text{ are integers and } m \leq n$$

The function tells us to take the number ‘$m$’ and perform $f(i)$ where $i = m$, so $f(m)$. Then, to that sum, add the number $f(m + 2)$, and so on, until we have added the number $f(n)$.

Examples:

$$\sum_{i=1}^{5} (i) = 1 + 2 + 3 + 4 + 5 = 15$$

$$\sum_{i=1}^{5} (i + 1) = (1 + 1) + (2 + 1) + (3 + 1) + (4 + 1) + (5 + 1) = 20$$

$$\sum_{i=2}^{4} a_i = a_2 + a_3 + a_4$$