AN INTRODUCTION TO

Probability and Inductive Logic

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done any inductive logic yet, but you probably think some of (7a)-(7d) are better arguments than others. Which is best? Which is worst?

**KEY WORDS FOR REVIEW**

- Argument
- Proposition
- True-or-false
- Premise

- Conclusion
- Valid
- Sound
- Conditional

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## 2 What Is Inductive Logic?

Inductive logic is about risky arguments. It analyses inductive arguments using probability. There are other kinds of risky arguments. There is inference to the best explanation, and there are arguments based on testimony.

Valid arguments are risk-free. Inductive logic studies risky arguments. A risky argument can be a very good one, and yet its conclusion can be false, even when the premises are true. Most of our arguments are risky.

Begin with the big picture. The Big Bang theory of the origin of our universe is well supported by present evidence, but it could be wrong. That is a risk.

We now have very strong evidence that smoking causes lung cancer. But the reasoning from all that evidence to the conclusion "smoking causes lung cancer" is still risky. It might just turn out that people predisposed to nicotine addiction are also predisposed to lung cancer, in which case our inference, that smoking causes lung cancer, would be in question after all.

After a lot of research, a company concludes that it can make a profit by marketing a special left-handed mouse for personal computers. It is taking a risk.

You want to be in the same class as your friend Jan. You reason that Jan likes mathematics, and so will take another logic class. You sign up for inductive logic. You have made a risky argument.

**ORANGES**

Here are some everyday examples of risky arguments.

A small grocer sells her old fruit at half-price. I want a box of oranges, cheap. But I want them to be good, sweet, and not rotten. The grocer takes an orange from the top of a box, cuts it open, and shows it to me. Her argument is:
(A) This orange is good.
So:
All (or almost all) the oranges in the box are good.

The premise is evidence for the conclusion: but not very good evidence. Most of the oranges in the box may be rotten.

Argument (A) is not a valid argument. Even if the premise is true, the conclusion may be false. This is a risky argument.

If I buy the box at half-price on the strength of this argument, I am taking a big risk. So I reach into the box, pick an orange at random, and pull it out. It is good too. I buy the box. My reasoning is:

(B) This orange that I chose at random is good.
So:
All (or almost all) the oranges in the box are good.

This argument is also risky. But it is not as risky as (A).

Julia takes six oranges at random. One, but only one, is squishy. She buys the box at half-price. Her argument is:

(C) Of these six oranges that I chose at random, five are good and one is rotten.
So:
Most (but not all) of the oranges in the box are good.

Argument (C) is based on more data than (B). But it is not a valid argument. Even though five out of six oranges that Julia picked at random are fine, she may just have been lucky. Perhaps most of the remaining oranges are rotten.

SAMPLES AND POPULATIONS

There are many forms of risky argument. Arguments (A)–(C) all have this basic form:

Statement about a sample drawn from a given population.
So:
Statement about the population as a whole.

We may also go the other way around. I might know that almost all the oranges in this box are good. I pick four oranges at random to squeeze a big glass of orange juice. I reason:

All or almost all the oranges in this box are good.
These four oranges are taken at random from this box.
So:
These four oranges are good.

This too is a risky argument. I might pick a rotten orange, even if most of the oranges in the box are fine. The form of my argument is:

Statement about a population.
So:
Statement about a sample.

We can also go from sample to sample:

These four oranges that I chose at random are good.
So:
The next four oranges that I draw at random will also be good.

The basic form of this argument is:

Statement about a sample.
So:
Statement about a new sample.

PROPORTIONS

We can try to be more exact about our arguments. These are small juice oranges, 60 to the box. A cautious person might express "almost all" by "90%," and then the argument would look like this:

These four oranges, that I chose at random from a box of 60 oranges, are good.
So:
At least 90% (or 54) of the oranges in the box are good.

At least 90% (or 54) of the oranges in this box are good. These four oranges are taken at random from this box.
So:
These four oranges are good.

PROBABILITY

Most of us are happy putting a "probably" into these arguments:

These four oranges, that I chose at random from a box of 60 oranges, are good.
So, probably:
At least 90% (or 54) of the oranges in the box are good.

At least 90% (or 54) of the oranges in this box are good.
These four oranges are taken at random from this box.
So, probably:
These four oranges are good.

These four oranges, that I chose at random from a box of 60 oranges, are good.
So, probably:
The next four oranges that I draw at random will also be good.
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Can we put in numerical probability values? That would be one way of telling which arguments are riskier than others. We will use ideas of probability to study risk.

Probability is a fundamental tool for inductive logic.

We will only do enough probability calculations to make ideas clear. The focus in this book is on the ideas, not on the numbers.

DEDUCING PROBABILITIES

Inductive logic uses probabilities. But not all arguments using probabilities are inductive. Not all arguments where you see the word "probability" are risky. Probability can be made into a rigorous mathematical idea. Mathematics is a deductive science. We make deductions using probability. In chapter 6 we state basic laws, or axioms, of probability. We deduce other facts about probability from these axioms.

Here is a simple deduction about probabilities:

- This die has six faces, labeled 1, 2, 3, 4, 5, 6.
- Each face is equally probable. (Each face is as likely as any other to turn up on a roll of the die.)
- So, the probability of rolling a 4 is 1/6.

This argument is valid. You already know this. Even if you have never studied probability, you make probabilities add up to 1.

You intuitively know that when two events are mutually exclusive—the die can land only one face up on any roll—and exhaustive—the die must land with one of the six faces up—then the probabilities add up to 1.

Why is the argument valid? Given the basic laws of probability, whenever the premises of an argument of this form are true, then the conclusion must be true too.

Here is another valid argument about probability:

- This die has six faces, labeled 1, 2, 3, 4, 5, 6.
- Each face is equally probable.
- So, the probability of rolling a 3 or 4 is 1/3.

Even if you have never studied probability, you know that probabilities add up. If two events are mutually exclusive—one or the other can happen, but not both

at the same time—then the probability that one or the other happens is the sum of their probabilities.

Given the basic laws of probability, whenever the premises of an argument of this form are true, then the conclusion must be true too. So the argument is valid.

The two arguments just stated are both valid. Notice how they differ from this one:

- This die has six faces, labeled 1, 2, 3, 4, 5, 6.
- In a sequence of 227 rolls, a 4 was rolled just 38 times.
- So, the probability of rolling a 4 with this die is about 1/6.

That is a risky argument. The conclusion might be false, even with true premises. The die might be somewhat biased against 4. The probability of rolling a 4 might be 1/6. Yet by chance, in the last 227 rolls we managed to roll 4 almost exactly 1/6 of the time.

ANOTHER KIND OF RISKY ARGUMENT

Probability is a fundamental tool for inductive logic. But we have just seen that:

- There are also deductively valid arguments about probability.

Likewise:

- Many kinds of risk arguments need not involve probability.

There may be more to a risky argument than inductive logic. Inductive logic does study risky arguments—but maybe not every kind of risky argument. Here is a new kind of risky argument. It begins with somebody noticing that:

- It is very unusual in our university for most of the students in a large elementary class to get As. But in one class they did.

That is odd. It is something to be explained. One explanation is that the instructor is an easy marker.

- Almost all the students in that class got As.
- So, the instructor must be a really easy marker.

Here we are not inferring from a sample to a population, or from a population to a sample.

We are offering a hypothesis to explain the observed facts. There might be other explanations. Almost all the students in that class got As.

- So, that was a very gifted class.
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So:
The instructor is a marvelous teacher.
So:
The material in that course is too easy for well-prepared students.

Each of these arguments ends with a plausible explanation of the curious fact that almost everyone in the class got an A grade.
Remember argument (*3) on page 2:
(*3) If James wants a job, then he will get a haircut tomorrow.
James will get a haircut tomorrow.
So:
James wants a job.

This is an invalid argument. It is still an argument, a risky argument. Let us have some more details. James gets his hair cut once in a blue moon. He is broke. You hear he is going to the barber tomorrow. Why on earth? Because he wants a job. The conclusion is a plausible explanation.

INFERENCE TO THE BEST EXPLANATION
Each of the arguments we’ve just looked at is an inference to a plausible explanation.
If one explanation is much more plausible than any other, it is an inference to the best explanation.
Many pieces of reasoning in science are like that. Some philosophers think that whenever we reach a theoretical conclusion, we are arguing to the best explanation. For example, cosmology was changed radically around 1967, when the Big Bang theory of the universe became widely accepted. The Big Bang theory says that our universe came into existence with a gigantic “explosion” at a definite date in the past. Why did people reach this amazing conclusion? Because two radio astronomers discovered that a certain low “background radiation” seems to be uniformly distributed everywhere in space that can be checked with a radio telescope. The best explanation, then and now, is that this background radiation is the result of a “Big Bang.”

“ABDUCTION”
One philosopher who thought deeply about probability was Charles Sanders Peirce (1839–1914). Notice that it is spelled “Peirce.” His name is not “Pierce.” Worse still, his name is correctly pronounced “purse.” He came from an old New England family that spelled their name “Pers” or “Perse.”

Peirce liked things to come in groups of three. He thought that there are three types of good argument: deduction, induction, and inference to the best explanation. Since he liked asymmetries, he invented a new name for inference to the best explanation. He called it abduction. So his picture of logic is this.

What Is Inductive Logic?

Deduction
Induction
Abduction

Induction and abduction are, in his theory, two distinct types of risky argument.
Some philosophers believe that probability is a very useful tool in analyzing arguments to the best explanation. Other philosophers, like Peirce, do not think so. There is a debate about that. We leave that debate to philosophers of science. The issues are very interesting, but this book will not discuss inference to the best explanation.

TESTIMONY
Most of what you believe, you believe because someone told you so.
How reliable are your parents? Your psychology instructor? The evening news? Believing what they say involves risky arguments.
I know I was born on February 14, because my mother told me so.
So:
I was born on February 14.

My psychology instructor says that Freud was a fraud, and is a worthless guide to human psychology.
So:
Freud is a worthless guide to human psychology.

According to the evening news, the mayor is meeting with out-of-town officials to discuss the effect of the flood.
So:
The mayor is meeting with out-of-town officials to discuss the effect of the flood.

These are risky arguments. The evening news may be misinformed. Your psychology instructor may hate Freud, and be a very biased informant.
The argument about your birthday is the least risky. It is still risky. How do you know that your parents are telling the truth?
You look at your birth certificate. You can’t doubt that! Well, maybe your parents lied by a day, so they could benefit from a new law about child benefits that took effect the day after you were born. Or maybe you were born on Friday the thirteenth, and they thought it would be better if you thought you were born on Valentine’s Day. Or maybe you were born on a taxi ride to the hospital, and in the excitement no one noticed whether you were born before or after midnight . . .

All the examples are arguments based on the testimony of someone else: your family, your instructor, the evening news.
Some kinds of testimony can be analyzed using probability, but there are a lot of problems. Inductive logic touches on testimony, but there is a lot more to testimony than probability.

In this book we will not discuss inference to the best explanation, and we will not discuss testimony. But if you really want to understand risky arguments, you should think about testimony, and inference to the best explanation. In this book, we study only one side of probability.

ROUGH DEFINITION OF INDUCTIVE LOGIC

Inductive logic analyzes risky arguments using probability ideas.

ROUGH DEFINITION OF DECISION THEORY

Decision theory analyzes risky decision-making using ideas of probability and utility.

EXERCISES

1 Fees. With a budgetary crisis, administrators at Memorial University state that they must either increase fees by 35% or increase class sizes and limit course offerings. Students are asked which option they prefer. There is a sharp difference of opinion.

Which of these risky arguments is from sample to population? From population to sample? From sample to sample?
(a) The student body as a whole is strongly opposed to a major fee increase. 65 students will be asked about the fee increase.
So:
Most of the 65 students will say that they oppose a major fee increase.

(b) A questionnaire was given to 40 students from all subjects and years. 32 said they were opposed to a major fee increase.
So:
Most students are opposed to a major fee increase.

(c) The student body as a whole is strongly opposed to a major fee increase.
So (probably):
The next student we ask will oppose a major fee increase.

(d) A questionnaire was given to 40 students from all subjects and years. 32 said they were opposed to a major fee increase.
So (probably):
The next student we ask will oppose a major fee increase.

2 More fees. Which of these is an inference to a plausible explanation? Which is an inference based on testimony?
(a) The student body as a whole is strongly opposed to a major fee increase.
So:
They prefer to save money rather than get a quality education.

(b) The student body as a whole is strongly opposed to a major fee increase.
So:
Many students are poor, and loans are so hard to get, that many students would have to drop out of school if fees went up.

(c) Duodecimal Research Corporation polled the students and found that 46% are living below the official government poverty line.
So:
The students at Memorial cannot afford a major fee increase.

3 Look back at the Odd Questions on pages xv-xvii. Each question will be discussed later on. But regardless of which answer is correct, we can see that any answer you give involves an argument.

3.1 Boys and girls. Someone argues:

About as many boys as girls are born in hospitals.
Many babies are born every week at City General.
In Cornwall, a country town, there is a small hospital where only a few babies are born every week.
An unusual week at a hospital is one where more than 55% of the babies are girls, or more than 55% are boys.
An unusual week occurred at either Cornwall or City General last week.
So:
The unusual week occurred at Cornwall Hospital.

Explain why this is a risky argument.
3.2 Pia. The premises are as stated in Odd Question 2. Which is the riskier conclusion, given those premises?
(a) Pia is an active feminist.
(b) Pia is a bank teller and an active feminist who takes yoga classes.

3.3 Lottery. Your aunt offers you as a present one of two free Lotto 6/49 tickets for next week's drawing. They are:
A. 1, 2, 3, 4, 5, and 6.
B. 39, 36, 32, 21, 14, and 3.
(a) Construct an argument for choosing (A). If you think it is stupid to prefer (A) over (B), then you can produce a bad or weak argument! But try to make it plausible.
(b) You decide to take (A). Is this a risky decision?

3.4 Dice.
Two dice are fair; each face falls as often as any other, and the number that falls uppermost with one die has no effect on the number that falls uppermost with the other die.
So:
It is more probable that 7 occurs on a throw of these two dice, than 6.
Is this a risky argument?

3.5 Testifies. Amos and Daniel are both jurors at a trial. They both hear the same information as evidence, namely the information stated in Odd Question 5. In the end, they have to make a judgment about what happened.
Amos concludes: So, the sidewiper was blue.
Daniel concludes: So, the sidewiper was green.
(a) Are these risky arguments?
(b) Could you think of them as risky decisions?

3.6 Step throat. The physician has the information reported in Odd Question 6. She concludes that the results are worthless, and sends out for more tests.
Explain why that is a risky decision.

4 Ludwig van Beethoven.
(a) What kind of argument is this? How good is it?
Beethoven was in tremendous pain during some of his most creative periods—pain produced by cirrhosis of the liver, chronic kidney stones (passing a stone is excruciatingly painful), and bouts of nonstop diarrhea. Yet his compositions are profound and often joyous.
So:
He took both pain killers and alcohol, and these drugs produced states of elation when he did his composing.
(b) Give an example of a new piece of information which, when added to the premises, strengthens the argument.
Books on "critical thinking" teach you how to analyze real-life complicated arguments. Among other things, they teach you how to read, listen, and think critically about the things that people actually say and write. This is not a book for critical thinking, but it is worth looking at a few real-life arguments. All are taken from a daily newspaper.

5 The slender eelfish.
A rare deep-sea creature, the slender eelfish, is helping Japanese scientists predict major earthquakes. In Japanese folklore, if an eelfish, which normally lives at depths of more than 200 meters, is landed in nets, then major tremors are not far behind.
Two slender eelfish were caught in fixed nets recently only days before a series of earthquakes shook Japan. This reminds us that one of these fish was caught two days before a major earthquake hit Yilimana Island, near Tokyo, in 1963. Moreover, when shock waves hit Uwajima Bay in 1968, the same type of rare fish was caught.
The eelfish has a unique elongated shape, which could make it susceptible to underwater shock waves. It may be stunned and then float to the surface. Or the real reason could be that poisonous gases are released from the Earth's crust during seismic activity. At any rate, whenever an eelfish is netted, a geological upheaval is in progress or about to occur.
And, having just caught some slender eelfish, Japanese seismologists are afraid that another disaster is imminent.
(a) In the first paragraph, there is a statement based on testimony. What is it? On what testimony is it based?
(b) The third paragraph states one conclusion of the entire discussion. What is the conclusion?
(c) The second paragraph states some evidence for this conclusion. Would you say that the argument to the conclusion (a) is more like an argument from population to sample, or from sample to population?
(d) The third paragraph offers two plausible explanations for the facts stated in the second paragraph. What are they?
(e) There are several distinct arguments leading to the final conclusion in the fourth paragraph. Describe how the arguments fit together.

6 Women engineers.
Since 1986, only 11% of engineering school graduates have been women. That showing is particularly poor considering that in other formerly male-dominated fields there are signs of real progress. Some eamples from 1986 include 48% commerce, 44% medicine, 45% and in the biological sciences, nearly 50% of the graduates are women.
(a) What is the conclusion? (b) What kind of argument is it? Valid? Inductive and risky? Inference to a plausible explanation?

7 Plastic surgery.
In her private counseling service for women, Martha Laurence, a professor of social work, sums up the reasons women give for wanting plastic surgery: "Unusually it is because they have a lack of confidence in who they are, the way they are," she said. "There is no simple answer, but the real problem is one of equity and of women's control over the self."
Her conclusion is that "the real problem is one of equity and of women's control over the self." What type of argument does she have for this conclusion?
3 The Gambler’s Fallacy

Most of the main ideas about probability come up right at the beginning. Two major ones are independence and randomness. Even more important for clear thinking is the notion of a probability model.

ROULETTE
A gambler is betting on what he thinks is a fair roulette wheel. The wheel is divided into 38 segments, of which:
- 18 segments are black.
- 18 segments are red.
- 2 segments are green, and marked with zeroes.

If you bet $10 on red, and the wheel stops at red, you win $20. Likewise if you bet $10 on black and it stops at black, you win $20. Otherwise you lose. The house always wins when the wheel stops at zero.

Now imagine that there has been a long run—a dozen spins—in which the wheel stopped at black. The gambler decides to bet on red, because he thinks:

- The wheel must come up red soon.
- This wheel is fair, so it stops on red as often as it stops on black.
- Since it has not stopped on red recently, it must stop there soon. I’ll bet on red.

The argument is a risky one. The conclusion is, “The wheel must stop on red in the next few spins.” The argument leads to a risky decision. The gambler decides to bet on red. There you have it, an argument and a decision. Do you agree with the gambler?

Since this chapter is called “the gambler’s fallacy” there must be something wrong with the gambler’s argument. Can you say what?
“All three could easily replicate themselves creating a cascade effect that could sweep through the physical world. . . . It is no exaggeration to say we are on the cusp of the further perfection of extreme evil.”

“The cusp of the further perfection of extreme evil”? Jill Kilroy has no idea what that means, but it sounds like very, very bad news. So she tries to construct an argument from dominance to show that we should stop work on robotics, genetic engineering, and nanotechnology, now.

Present Jill K’s argument briefly and precisely, with utility assignments and a simple partition of possibilities. Include a decision table. Then present general criticisms of the argument.

**KEY WORDS FOR REVIEW**

- Decision under uncertainty
- Pascal’s wager
- Partition
- Live possibilities

  **Dominance**
  - Causal influence
  - Dominance rule
  - Dominant expected value rule

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**11 What Do You Mean?**

The idea of probability leads in two different directions: belief and frequency. Probability makes us think of the degree to which we can be confident of something uncertain, given what we know or can find out. Probability also makes us think of the relative frequency with which some outcome occurs on repeated trials on a chance setup.

Thus far we have used both ideas almost interchangeably, because the basic rules for calculating with each are virtually identical. But now we have to distinguish between them, because the philosophical and practical uses of these two ideas are very different. The distinction is essential for the rest of this book (and for all clear thinking about probability).

We have been doing all these calculations about probabilities, and have not said a word about what we mean by “probability” itself. Now we are going to set things right. Up to now it has not mattered a great deal what we mean by the word. From now on it will make all the difference.

This chapter is an example of one kind of philosophy, often called analytic philosophy. We will try to come to grips with different concepts associated with the idea of probability. Many students find this chapter the hardest one of all. Not surprising! The distinctions that we have to make have bedeviled probability theorists—including some of the very best—for more than 200 years. Debates between experts who take the frequency approach, and those who take the belief approach, continue to be acrimonious. In this chapter and the next, you will have to make up your own mind about where you stand on these issues.

**A BIASED COIN**

In ordinary language we use the word “probability” in different ways. We say things like this:
(1) This coin is biased toward heads. The probability of getting heads is about 0.6.

We take for granted that (1) is a statement about the coin. It implicitly refers to some method of tossing in a definite chance setup. Here are five remarks about (1):

a. Statement (1) is either true or false, regardless of what we know about the coin. If (1) is true, it is because of how the world is, especially how the coin and tossing device are.

b. If (1) is true, we suppose that the coin is asymmetrical. (Or maybe there is something unusual about the tossing device.)

c. We imagine that someone could explain (1) by facts drawn from the geometry of the coin, or the laws of physics.

d. We could do experiments to test (1). We conduct many trials on the chance setup, and observe the relative frequency of heads. If we get 63 heads irregularly distributed in 100 tosses, we are ready to accept (1) as true. But if we get only 37 heads, we are very dubious about (1).

e. In short: (1) states a fact about how the world is, and we can collect evidence to see whether (1) is true or false.

Notice that we could say very much the same thing about radioactive decay. If you put a Geiger counter next to a small piece of radium, you hear clicks every time a particle of radioactive decay passes through the detector. Holding the counter four feet away from the source, you will notice a random sequence of clicks. This is a chance setup, and you might find out that:

The probability of getting a click in any given three-second interval is 0.6.

a. This statement is true or false, regardless of what we know about the radium and the detector.

b. We imagine that someone could explain it by facts drawn from the way the counter is made, facts about the sample of radium, and the laws of physics.

c. We could do experiments to test the statement. We observe the relative frequency of three-second periods in which there are clicks.

d. In short: the statement states a fact about how the world is, and we can collect evidence to see whether it is true or false.

More sophisticated statements about the half-life of radium, for example, are to be understood in the same way.

THE EXTINCT DINOSAURS
We also say things like this:

(2) It is probable that the dinosaurs were made extinct by a giant asteroid hitting the Earth.

What Do You Mean?

We can add details and precision to (2), as in this conversation from the TV program "The Science of Yesterday":

Science journalist Betty Glossop: There is a lot of new evidence about a layer of iridium deposits in many parts of the Earth. Geologists have identified them as contemporary with the extinction of the dinosaurs.

The interviewer for Penchant: What's that got to do with it?

Betty: Iridium is an uncommon element that is the most corrosion-resistant substance found in nature. We have identified an asteroid crater rich in iridium, presumably from the asteroid. We think that it produced a gigantic cloud of dust, including iridium, that covered the earth. Plants needed for vegetation, dinosaurs, and for the prey of carnivorous ones, simply didn't grow well and the dinosaurs starved to death.

Jef: So how probable is it, in the light of all this new information, that it was an asteroid that killed off the dinosaurs?

(3) Betty: Taking all the evidence into consideration, the probability is about 90%.

The word "probability" seems to be used differently in (1) and (3). In (3), Betty Glossop is talking about the probability of a proposition, namely:

(4) The dinosaurs were made extinct by a giant asteroid hitting the Earth.

Let us first of all look just at (3), where Betty refers to her evidence about iridium and so on. Let's make some contrasts, item by item, between (3) and points a-e about (1), the biased coin. Read the following statements, and the previous one, very carefully. See if you agree with every single statement.

First, review exactly what (1), (2), (3), and (4) are. Notice that (4) is not a probability statement at all. Notice that only (3) mentions evidence.

DIINOSAURS AND PROBABILITY

a. Statements (1) and (4) [but not (3)] are similar in one respect. Statement (4), like (1), is either true or false, regardless of what we know about the dinosaurs. If (4) is true, it is because of how the world is, especially what happened at the end of the dinosaur era. If (3) is true, it is not true because of "how the world is," but because of how well the evidence supports statement (4).

b. If (3) is true, it is because of inductive logic, not because of how the world is.

c. The evidence mentioned in (3) will go back to laws of physics (iridium), geology (the asteroid), geophysics, climatology, and biology. But these special sciences do not explain why (3) is true. Statement (3) states a relation between the evidence provided by these special sciences, and statement (4), about dinosaurs.

d. We can do experiments to test the claims about iridium and so on. But we cannot do experiments to test (3). Notice that the tests of (3) may involve
repeated tosses of the coin. But it makes no sense at all to talk about repeatedly testing (3).

e. In short: statement (3) makes a claim about how the evidence supports statement (4).

ON NOT MENTIONING EVIDENCE

We have supposed (2), which is like (3), but does not mention the evidence. To make (2) look more like (1) and (3), we will put a number on that "probable":

(2.1) The probability that the dinosaurs were made extinct by a giant asteroid hitting the Earth is very high—about 0.9.

Statement (2.1) is different from (3), because it does not mention evidence. But in some ways it is like (3). Point d is one way they are alike:

d. We can do experiments to test the claims about iridium and so on, which might lead someone to assert (2.1). But we cannot do experiments to test (2.1). Notice that the tests of (1) may involve repeated tosses of the coin. But it makes no sense at all to talk about repeatedly testing (2.1)

Unfortunately, there are at least two ways to understand (2.1). We may think that (2.1) is "really" just short for (3). When people say that so and so is probable, they mean that relative to the available evidence, so and so is probable. We will call this the interpersonal/evidential way to understand (2.1). We will call the other way to understand (2.1) the personal way.

INTERPERSONAL/EVIDENTIALLY

On this way of understanding (2.1), it is short for:

(2.2) Relative to the available evidence, the probability that the dinosaurs were made extinct by a giant asteroid hitting the Earth is very high—about 0.9.

We could say of someone who long-windedly states (2.2):

• She is taking for granted that any reasonable person who thought about the evidence, would find it very reasonable to think that an asteroid wiped out the dinosaurs.
• She thinks that it is rational to be pretty confident that the asteroid caused the extinction of the dinosaurs.
• She thinks that to say (2.1) is to mean something like (2.2)—or Betty Glassop's less long-winded (3).
• She thinks that (2.2) is interpersonal—because it is about what it is reasonable for any reasonable person to believe. And since the degree of belief should depend on the available evidence, we call this interpersonal/evidential.

• Since she thinks that (2.2) is interpersonal, and about rational degrees of belief, she thinks that (2.2) is "objective."

SUBJECTIVE/OBJECTIVE—NOT

If you read a lot about probability, you will often read about "objective" and "subjective" probabilities. These are terrible terms, loaded with ideology. "I'm objective, you're subjective, he is prejudiced." How often have you heard this sort of conversation?

JAMES: That's just your subjective opinion.
MARY: Nonsense, it is an objective fact.

How often have you talked just like that?

Don't get into that rut, in probability or in the rest of your life. James and Mary are not arguing, they are just slingin' mud at each other.

But do notice that both (1), about the bias and the die, and (2.2) or (3), can be called "objective"—although for quite different reasons. Statement (1) is called "objective" because it is a statement about how the world is. Statement (2.2) is called "objective" because of a supposed logical relation between the evidence and a proposition [namely, (4)].

PERSONAL DEGREE OF BELIEF

There is another way to understand (2). When someone says (2), they may mean only something about themselves, something like:

(2.3) I personally am very confident that the dinosaurs were made extinct by a giant asteroid hitting the Earth.

Or even this way to understand (2.1):

(2.4) If I had to make a bet on it, I would bet 9 to 1 that the dinosaurs were made extinct by a giant asteroid hitting the Earth.

TIME TO THINK ABOUT YOURSELF

What do you mean, when you say things like (2), "It is probable that the dinosaurs were made extinct by a giant asteroid hitting the Earth?"

What do you mean when you say things like:

It will probably rain today.
It is probable that I will flunk my geology test.
I just can't tell those stupid rock samples one from the other.

The probability that there was a second gunman involved in the John F. Kennedy assassination is negligible.
In all probability, we are going through a period of extreme global warming due to burning fossil fuels, aerosols, methane produced by manure from beef cattle grown so we can kill and eat them, and so on. Despite all the propaganda, the global warming hypothesis is not at all probable; we are just going through a routine climatic cycle. It is very probable that the company my father works for will be downsized after the takeover, and he will be out of a job.

**BELIEF-TYPE**

Statement (4) was a proposition about dinosaur extinction; (2)-(2.2) and (3) are about how credible (believable) (4) is. They are about the degree to which someone believes, or should believe, (4). They are about how confident one can or should be, in the light of that evidence.

The use of the words "probable" and "probability" in (2)-(2.2) and (3) is related to ideas such as:

- belief
- confidence
- credibility
- evidence
- propensity
- frequency
- tendency
- disposition
- propensity
- symmetry

We need a name for this family of uses of probability words. Philosophers have used a lot of different names. The easiest to remember is:

**BELIEF-TYPE PROBABILITY**

This is not the end of the matter. We just saw that belief-type probabilities can be thought of in at least two ways: *interpersonal/evidential and personal.*

**FREQUENCY-TYPE**

Now look at (1) again:

(1) The probability of getting heads with this coin is 0.6.

The truth of this statement seems to have nothing to do with what we believe. We seem to be making a completely factual statement about a material object, namely the coin (and the device for tossing it). We could be simply wrong, whether we know it or not. This might be a fair coin, and we may simply have been misled by the small number of times we tossed it. We are talking about a physical property of the coin, which can be investigated by experiment.

What is this physical property? We may be saying something like:

**What Do You Mean?**

- In repeated tossing, the relative frequency of heads settles down to a stable proportion, 6/10.
- The coin has a tendency to come down heads far more often than tails.
- It has a propensity or disposition to favor heads.
- Or we are saying something more basic about the asymmetry of the coin and tossing device. We may be referring to the geometry and physics of the coin, which cause it to come down more often heads than tails.

The use of the word "probability" in (1) is related to ideas such as:

**OTHER NAMES**

You won't believe the number of names that philosophers have given to these two groups of uses of the word "probability." In case you look at other books, here is a little dictionary.

- Subjective/objective. The oldest pair of words for the belief-type and frequency-type distinction is "subjective" (belief-type) and "objective" (frequency-type). Objection: why say that the long statement (3), referring to the available evidence about the iridium layer and so forth, is "subjective"? Many scientists would claim that it is an "objective" assessment of the evidence.
- *It's all Greek to me.* Belief-type probabilities have been called "epistemic"—from *episteme,* a Greek word for knowledge. Frequency-type probabilities have been called "aleatory," from *alou,* a Latin word for games of chance, which provide clear examples of frequency-type probabilities. Objection: these words have never caught on. And it is much easier for most of us to remember plain English words rather than fancy Greek and Latin ones.
- Number 1 and number 2. The philosopher and logician Rudolf Carnap (1891-1970) called belief-type probability "probability," and frequency-type probability "probability." Objection: Carnap's proposal never caught on. And it is hard to remember which is number 1 and which is number 2.

Many other labels for the two groups of probability ideas have been used, but that is enough for now. Now let us look at some of our own examples earlier in
the book. Where did we implicitly think of frequency-type probability? Where did we implicitly think of belief-type probability?

SHOCKS

Page 52 gave some data about two suppliers of shock absorbers, Bolt & Co. and Acme Inc. We began with the information that:

Bolt supplies 40% of the shock absorbers for a manufacturer, and Acme 60%. We took the probability of a randomly selected shock absorber being made by Bolt to be 0.4.

Of Acme’s shocks, 96% test reliable. But Bolt has been having some problems on the line, and recently only 72% of Bolt’s shock absorbers have tested reliable.

We asked:

What is the probability that a randomly chosen shock absorber will test reliable? The probability worked out to be 0.864.

What is the conditional probability that a randomly chosen shock absorber, which is tested and found to be reliable, is made by Bolt? The answer was 0.5.

Here it is natural to take a frequency perspective.

We are talking about a mass-produced product from an assembly line. We observe the relative frequency of defective and reliable products from the two lines, one at Bolt, one at Acme. These frequencies must reflect a difference between the two companies.

STREP THROAT

Page 75 discussed Odd Question 6, about strep throat. You were asked to imagine that you were a physician examining a patient, and sending swabs to the lab for testing. We said,

You think it likely that the patient has strep throat. Let us, to get a sense of the problem, put a number to this, the probability is 90% that the patient has strep throat. Pr(S) = 0.9.

You, as physician, were asked to reach a conclusion on the basis of seemingly inconsistent reports from the lab: 3 positive tests and 2 negative ones. You concluded,

It is very much more likely than not, that the patient does have strep throat. The probability that the patient has strep throat, given the data, is 343/344, or 0.997.

Then we moved to another case, called “street ignorance.” We said that the ignorant person might say:

It is 50-50 whether this patient has strep throat.

The probability that the patient has strep throat is 0.5.

That is, Pr(S) = 0.5.

You computed your probability of strep throat, in the light of the news from the lab, as the rather surprising 343/352, or 0.974.

In this example we do have some frequency data, namely the probability of false positives in the lab test. Nevertheless, we are plainly talking about the beliefs of the physician or the ignorant amateur, both before and after getting the lab results.

In this case it is natural to take a belief perspective.

Notice that the problem about shock absorbers and the problem about strep throat can be solved in exactly the same way, using Bayes’ Rule. The formal, logical, arithmetical problem is the same, but the meaning is somewhat different.

STATING THAT, AND REASONS FOR

Frequency-type probability statements state how the world is. They state, for example, a physical property about a coin and tossing device, or the production practices of Acme and Bolt.

Belief-type probability statements express a person’s confidence in a belief, or state the credibility of a conjecture or proposition in the light of evidence.

Beware of a confusion that troubles a lot of students.

If I state a matter of fact, you expect me to believe what I state. You expect me to have some reasons for thinking my belief is true. You expect me to be able to give you my reasons.

So some students think: every statement about how the world is, states reasons and beliefs. NO!

A statement about how the world is, is a statement made (we hope) because a person has some beliefs and some reasons for them. But what it says is not, “I have reasons for my belief that p.” What it says is, “This is how the world is p.”

Distinguish:

♦ What a person says (what a proposition states).
♦ The reasons that a person may have, for stating or believing some proposition.

Actually, people often do not have reasons for what they say. Sometimes they do not even believe what they say. A person may say “how the world is” in many different circumstances. The person may:

♦ Have excellent reasons.
♦ Merely hope the statement is true but have no reasons.
**The Single Case**

Frequency-type probabilities usually are about items of some kind that occur in a sequence: spins of a wheel, shock absorbers produced by a manufacturer.

It does not make sense to speak of the "frequency" of a single event. A patient either has, or has not, got strep throat. In the taxicab problem, Odd Question 5, either a blue cab is side-swiped another car, or a green cab did it. As stated, these problems involve a single event, a single car, a single patient, a single case. Such probabilities cannot literally be understood as a frequency. They cannot be understood as tendencies or propensities either.

In the taxicab story, the witness has a tendency or propensity or disposition to make a correct identification of cab colors on a misty night. She got things right 80% of the time. That is a frequency. But there is not a tendency on the part of the bystander to be green or blue. It was green. Or it was blue.

If someone speaks of the probability of a single event, then they must be taking the belief perspective.

Sounds simple, but beware. Dean tosses a coin, and it falls on the table in front of me. Before Dean or anyone else has a chance to observe the outcome, he slams a book down on top of the coin.

DEAN: What is the probability that the coin under the book is heads up?

BEANCO: 60%.

DEAN: You mean that the probability is 0.6, that the coin under the book is heads up?

BEANCO: Yes.

DEAN: Why do you say that?

BEANCO: I thought you were tossing that biased coin you discussed in (1) at the start of this chapter. I thought that this coin is biased towards heads, and the probability of getting heads is about 0.6.

In this case, Beanco is making a belief-type statement about this toss of this coin, a single case. He has a reason for this belief-type statement, namely a frequency-type statement about the coin.

**Switching Back and Forth**

We use one word, "probability," from both a frequency and a belief perspective. That is no accident. We switch back and forth between the two perspectives. We computed that:

The conditional probability is $1/3$ that a shock absorber is made by Bolt, if it has been tested at random and found to be reliable.

That is a statement about the production characteristics of shock absorbers, bought by the automobile manufacturer. Tomorrow, Rosie the Reticent, the quality control engineer, tests a shock absorber at random from a batch of shocks. She finds that it is reliable.

What is the probability that this shock absorber was made by Bolt?

Rosie knows the answer: the probability is $1/3$.

But this probability is about a single case. There is no frequency with which this shock, or this batch, is made by Bolt. It is, or it is not.

Rosie made a belief-type statement. Her reason for making it was her knowledge of the relative frequencies with which randomly selected shock absorbers, in this setup, are reliable.

So Rosie has switched from a frequency perspective to a belief perspective.

**The Frequency Principle**

We switch perspectives by a rule of thumb, which has been called the frequency principle. It connects belief-type and frequency-type probabilities.

It is a rule about knowledge and ignorance. Suppose that:

- You know the frequency-type probability of an event on trials of some kind.
- You are ignorant of anything else about the outcome of a single trial of that kind.

Then you take the frequency-type probability as the belief-type probability of the single case.

We could call it a knowledge-not-knowing-belief-the-frequency rule. Here is a very pedestrian way to state the frequency principle:

If $S$ is an individual event of type $E$, and the only information about whether $E$ occurred on a trial of a certain kind, on a certain chance setup, is that on trials of that kind on that setup the frequency-type probability $Pr(E) = p$, then the belief-type probability of $S$ is also $p$.

**Relevant Subsets**

The frequency principle is a rule of thumb. When is a frequency-type probability absolutely "all" that we know about the occurrence of an event? Only in artificial situations, as when we toss a fair coin and hide the result. Nevertheless, in real life we quite often have "something like" this situation.

Usually we have a lot of not-very-tidy information. The weather office took the smallest subset of weeks like the week just ended, and worked out the tendency of such weeks to be followed by precipitation.

When we implicitly use the frequency principle, we often make a judgment
of relevance like that. Recall Tomer, the heavy smoker, from page 120. His friend Peggy was told by the statistician that:

The probability of a male of about Tomer's age (and otherwise like Tomer) dying before age sixty-five, given that he smokes, is 0.36.

The statistician chose a relevant subset of males, chiefly by age, but also with a view to getting a trustworthy statement of frequency-type probabilities. Real-life application of the frequency principle requires a lot of judgment.

**Probabilities of Probabilities**

Back to the gambler's fallacy. Alert Learner (page 31) noticed that the wheel had stopped at black on twelve spins in a row. She suspected bias. After doing some more experiments on the wheel, suppose she concludes that the wheel is heavily biased toward black:

\[ 0.91 \leq P(B) \leq 0.93. \]

She intends this to be understood as a statement about the properties of the wheel. Because of the way in which the wheel is made and spun, it stops at black about 92% of the time.

This is a risky conclusion. Alert Learner may still feel confident enough to say,

It is very probable that the probability of black is close to 0.92.

She might even try something more precise-sounding: "The probability that 0.91 \leq P(B) \leq 0.93 is at least 95%.

Apparently Alert Learner is expressing a belief-type probability of a frequency-type probability. How on earth can she do that? Easy! It is a matter of fact, whether the probability of getting black is between 0.91 and 0.93 or not. And we can discuss the probability of that being true. A probability (belief-type) of a probability (frequency-type).

**Exercises**

1. Shock absorbers again. (a) In the shock absorber example, Acme had a better record of reliability than Bolt. What might be the causes of this difference? (b) Why are causes relevant to the distinction between frequency-type and belief-type probabilities?

Classify each of the following statements in italics as frequency-type or belief-type. If you think that one or more could be understood as either frequency-type or belief-type, explain why.

2. Influenza. These are all quotations from a newspaper story.

(a) When a flu epidemic strikes, the probability that a person who was exposed will get sick is between 10% and 15%.

(b) The disease is likely to run its course in 5 to 7 days in healthy young people.

(c) But it is far more probable that the flu will last for weeks in an old person than in a healthy young person.

(d) Making a vaccine is always a guessing game, because there is no way to predict what flu strains will appear. But researchers can get a good idea by detecting what strains start cropping up at the end of a previous year. If a strain appears at the end of the season, there is a good probability that it will make the rounds next year.

(e) The probability that the flu vaccine prevents flu in a young healthy person is 70 to 90 percent, Dr. Gesa said. The probability is only 50 percent in people over 65. But, he added, the probability that it prevents death in an older person is 0.85.

3. (a) January 31, 1986 (from a news story that day): China is determined to abolish the local legislature... It is also probable that it will weaken Hong Kong status that protect civil liberties.

(b) February 1, 2006 (from a news story to appear that day): Contrary to recent reports, it is not probable that the breakaway southern province will choose Hong Kong as its capital city.

4. The Fed. The speaker was the chairman of the Federal Reserve Board (the "Fed") that determines the U.S. money supply and basic interest rates. According to a newspaper story, the chairman said:

(a) The probability of a recession is less than 50%, in contrast with growing fears of an economic downturn a year ago.

(b) Recent Fed analysis of leading economic indicators put the chances of a recession in the next 6 months even lower, at between 10 and 20 percent.

(c) I wouldn't bet the ranch on such statistical matters.

5. Crones (from Nature, one of the major weekly science journals). Researchers have identified a gene linked with Marfan syndrome—which involves a wide variety of problems, eyesight defects, heart disease and abnormally long limb bones... Francesco Ramirez has now cloned the gene for fibrillin and mapped it to a segment of chromosome 15. After studying families with the disorder, he concluded that the probability of exhibiting Marfan syndrome, given the defective gene, is over 0.7.

**Key Words for Review**

- Frequency
- Belief
- Interpersonal/evidential
- Single case
- Personal probability
- Frequency principle
- Propensity
- Relevant subset
Answers to the Exercises

4. Combinations. Only (b) is impossible. An argument with all premises true, and a false conclusion is, by definition, invalid.

5. Soundness. Only (a) is sound. Sound arguments are valid, and have true premises.

6. Conditional propositions. (a) is valid or invalid. It is an argument; (b) is true or false. It is a conditional, if-then proposition.

7. Clothing tobacco. All four arguments are invalid.

8. Inductive base. None of the four arguments is worthless. (7) is the strongest, assuming that the number of chewers and the number of nonchewers in 1956 were both quite large, and that the seven teams were chosen haphazardly, by the investigators. We might disagree about which of (7a), (7b), and (7d) is weakest. They can be criticized for different reasons. What is inductive logic is all about.

CHAPTER 2. WHAT IS INDUCTIVE LOGIC?

1. (a) Population to sample. (b) Sample to population. (c) Population to sample. (d) Sample to sample.

2. Same type (a) and (b) are inferences to a plausible explanation. (c) is an inference based on testimony.

3. Boys and girls. (a) The argument is risky because all the premises could be true while the conclusion is false.

3. (a) is the riskier conclusion. With (a) you are taking just one ride. Pia might not be an active feminist. But with (e) you are taking three risks—she might not be a bank teller, she might not be an active feminist, and she might not take yoga classes.

3. Lottery. (a) The probability that A wins is essentially the same as the probability that B wins. If two people choose the same combination of numbers, and that combination wins a big prize, they split the prize. People in general don't like regular-looking numbers so the probability that someone else picks A is less than the probability that someone else picks B. No if I bet on A and win, I'll get a much bigger share of the prize than if I bet on B and win. So I choose A. (b) This is not obviously a risky decision, since you lose nothing no matter what happens—the lottery tickets are free. But you do risk being regretful if you choose A and B wins.

4. Dice. No, this is not risky. It is a valid argument.

5. Taxes. (a) Yes (b) You could think of it as risky decisions, because both Amos and Daniel have to decide what to say to the judge or to their fellow jurors.

6. Sleep threat. The risk is that the patient does have strep throat, which will rapidly get worse if treatment is not commenced.

4. Ludwig von Beethoven. (a) Inference to a plausible explanation. How plausible? Musicologists are not impressed. (b) Many examples might do, for instance, new letters by Beethoven saying that he composes when he is high on opium. More interesting: A laboratory in Tenero is doing an analysis of a lock of 582 strands of Beethoven's hair, which should reveal trace elements of whatever chemicals the composer ingested.

5. The smaller. Carfish. (a) If an earfish, which normally lives in depths of more than 200 meters, is landed in nets, then major tumors are not far behind. It is based on the testimony of Japanese folklorists. (b) Conclusion: Whenever an earfish is netted, a geological upheaval is in progress or about to occur. (c) From sample to population— from individual earfish catches and quakes, a general statement about earfish and quakes. (d) Plausible explanation: The earfish, with its elongated shape, may be stunned and then float to the surface. Plausible explanation: Poisonous gases are released from the earth's crust during seismic activity. (e) First, there is an argument from testimony. Second, there is an inductive argument from sample to population. Third, there are proposals of plausible explanations, either of which, if true, would predict the conclusion. The conclusion is supported by three different types of risky argument.

6. Women engineers. (a) "That showing is particularly poor." (b) Valid (although the validity may depend on what we mean by "particularly poor.").


8. Manitoba marijuana. This resembles an argument from sample to population. The sample consists of homes that the police have checked for hydroponic marijuana. The population consists of rural homes in Manitoba.

CHAPTER 3. THE GAMBLER'S FALLACY

1. Roulette wheels. (a) In North America, the probability of the wheel stopping at red is 18/38. In Europe, the probability is 18/37. So if you are simply betting on red, Europe is better. (b) No.

2. Shuffling. (a) Yes. Every card should, in the course of a great many games, be dealt as often as every other. (b) No. (c) No. BUT: Since the players are unlikely to be able to predict the order of the cards after a shuffle, no one has an advantage despite the fact that the deal has a memory. Hence from the player's point of view the game is, in another sense of the word, "fair."

3. Lotto. It looks as if this is a good buy: if few people choose this sequence to bet on, then, if it does come up, they will have to share their winnings with fewer people than if they had bought a popular ticket. BUT how do I know this to be a fact? If I got it from a popular book on gambling, I can imagine that the book itself has changed people's habits, so that now, after the book has been around for a while, this is the most popular ticket?

4. Noncards. (a) Birth weights. In grams, we expect the last digit to be unbiased and independent of previous trials. But in pounds, 7 would occur more frequently than any other digit, so the set up would be biased. Trials would, however, still be independent (except for twins, or worse, quintuplets). (b) Telephone poles. Unbiased and independent. (c) Beds. 100% biased against 1, 3, 5, 7, and 9. (d) Cruise ships. Unbiased, independent.

5. Fallacious Gambler strikes back. (a) No. Fallacious Gambler sensibly supposes that "thirteen blacks" counts as a "long run," and concludes that, according to Dr. Marle's data a sequence of twelve blacks followed by a red occurs more often than a sequence of twelve blacks followed by a black. Of course when we dig up the records, it may turn out that Marle was referring to much longer runs, but at any rate Fallacious Gambler was not committing the gambler's fallacy.

(b) Here is one possible explanation: the croupiers who spin the roulette wheels think people will imagine the wheel is biased, and so think the house is dishonest. So after a long run, they do their best to spin the wheel so as to break the long run. Or the house itself deliberately puts a mechanical "corrector" behind the wheel, which, when the croupier pushes a button, biases the wheel toward black, when red has been appearing many times and vice versa.

6. Counting. Someone who can remember what cards have been dealt for previous hands has an advantage. For example, if you know that most of the low cards have already been dealt, then the probability of getting a high card is greater than it was at the beginning. Using this information, it is possible to improve the odds. You need quite a lot of capital, because you have to make large bets, and will sometimes lose. A mathematician named
CHAPTER 10. DECISION UNDER UNCERTAINTY

Some questions asked you to use your own utilities and probabilities. What follows are my judgments. They are not right or wrong. Your numbers will be different, but the resulting arguments should be similar.

1. Monetary value. Suppose Sarah's estimates of annual salary five years after graduation are as in this table:

<table>
<thead>
<tr>
<th>B</th>
<th>C</th>
<th>G</th>
</tr>
</thead>
<tbody>
<tr>
<td>30K</td>
<td>60K</td>
<td>45K</td>
</tr>
<tr>
<td>P</td>
<td>50%</td>
<td>50%</td>
</tr>
</tbody>
</table>

The $8,000 is what she thinks she will get on welfare. Neither act C nor act P has any causal influence on the economy, so the dominance rule can be applied. Sarah decides to take a degree in computer science, using the dominance rule.

2. Groom Penny. Penny has two different sources of utility: income and job contentment. She might represent her utilities this way:

<table>
<thead>
<tr>
<th>B</th>
<th>C</th>
<th>G</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>P</td>
<td>50%</td>
<td>50%</td>
</tr>
</tbody>
</table>

There is no dominating act. Here she requires probabilities of good and bad times. She is an optimist. She judges that

\[ P(B) = 0.3, \ P(G) = 0.7, \ P(C) = 0.8. \]

\[ \text{Exp}(C) = 5.4 \text{ utils,} \]

\[ \text{Exp}(P) = 0.8(5) + 0.2(7) = 7.3 \text{ utils.} \]

She applies the expected value rule, and decides to take a degree in philosophy.

3. Idealist Maria. Maria might represent her utilities in this way:

<table>
<thead>
<tr>
<th>B</th>
<th>C</th>
<th>G</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>70</td>
<td>50</td>
</tr>
<tr>
<td>P</td>
<td>100%</td>
<td>100%</td>
</tr>
</tbody>
</table>

Clearly act B dominates. Since neither act affects the economy, she applies the dominance rule and decides to take a philosophy degree.

4. Criticizing Pascal.

(a) The partition is wrong. There are obviously many more live possibilities, at least for us. There are many more religions. Buddhism has nothing like the reward values of Pascal's Catholicism. Someone might even think this is possible. God is malicious, and actually punishes those who believe in him if they have come to believe as a consequence of decision theory.

(b) This criticism is not applicable, for we do not think our personal acts of belief affect the existence or nonexistence of Pascal's God.

(c) The gambler challenges Pascal's claim that the utilities of not-belief are zero, and thus forces expected value considerations.

5. Study or not. No, because the decision affects the outcome. If he thought that

\[ P(A/B) = 0.2, \ P(B/R) = 0.6, \ P(C/R) = 0.2, \]

\[ P(A/S) = 0.7, \ P(B/S) = 0.3, \ P(C/S) = 0. \]

Then, weighing the pluses and minuses in the table as 1, 2, 3, etc., his expected values are:

\[ \text{Exp}(R) = 0.2(0) + 0.6(4) = 2.4, \]

\[ \text{Exp}(S) = 0.7(4) + 0.3(0) = 2.8. \]

So he should decide to study.

6. Twenty-first-century gleam. Let S = stop work on robotics, genetic engineering, nanotechnology, etc., now.

Let C = Carry on with that work.

Let T = Terrible results ensue from carrying on.

Let \( -T \) = There are no terrible results from carrying on.

There are many possible assignments of utilities which would yield an argument from dominance. Here is one:

<table>
<thead>
<tr>
<th>T</th>
<th>-T</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5</td>
</tr>
</tbody>
</table>

A high U(C,T) is just as bad as -w, and any utility assignment that leaves U(C,-T) no lower than U(S,-T) is fine.

The most obvious criticism is that if there are no terrible consequences, that the utility of carrying on will be greater than the utility of doing nothing, U(C,-T) is greater than U(S,-T), and so there is no sound argument from dominance. But there are other criticisms, for example, that there are more alternatives—you would have to spell out what alternatives you have in mind.

CHAPTER 11. WHAT DO YOU MEAN?

1. Shock absorbers again.
   (a) Perhaps Acme has union workers, who are well paid with excellent benefits, while Bolt uses nonunion labor and pays less well with no benefits. Maybe T is the opposite. Bolt is unionized and the workers have lost incentive, while at Acme you get fired for the least step-down. Maybe Acme has newer (or older) machines that are more trustworthy. Perhaps Acme has a better quality control engineer.
   (b) Whatever the cause, we think there is a definite tendency for Acme to produce more reliable shock absorbers than Bolt. We are concerned not with what anyone believes, but with some in-the-world fact about the two manufacturers.

2. Inference.
   (a) Frequency-type, or a statement about "the world," about the natural course of events under current conditions.
   (b) "Likely" here seems to mean that for most young people who catch the disease, it runs its course in a week or less. A frequency-type probability (7%?) seems to be implied.
   (c) Frequency-type.
   (d) This statement seems to be a claim about what can be expected to happen regularly under current conditions. So it seems to be frequentist-type, perhaps with a propensity.
   (e) All three statements are plainly frequency-type.

3. (a) January 31, 1996. Belief-type. The pre-testability statement is relative to the evidence available in 1996. We now have a lot more evidence.
   (b) February 2, 2006. Belief-type, relative to evidence available as of 2006.

   (a) Clearly a single case. Belief-type, intended to be imprecise, and perhaps relative to available evidence.
CHAPTER 12. THEORIES ABOUT PROBABILITY

1. Indifference. There is no "right" answer, for this is a matter of Matter’s personal probabilities. If he goes for a兴奋 instead of a bet, his personal probabilities for riches coming on that airline are greater than the sum of his personal probabilities for Alpha, plus his personal probability for Beta. But if it's personal probability for兴奋 is not that high, he should go to terminal B (or 2, depending on his beliefs about Gamma and Delta) rather than 3.

For variety, the answers to Exercises 2-4 give slightly different verbal expressions of the interpretations made by our theories.

2. Happy Harry.

Venn: The relative frequency of -successes, among students who use the kit, is 90%.

Pepper: There is a 90% tendency or propensity to produce success, using the kit.

De Finetti: Happy Harry (for his advertisement) are saying that their personal probability for success is 0.9—they'd bet 9 to 1 that any arbitrarily chosen student who uses the kit will succeed.

Keynes: Given the evidence that a student has used the kit, the logical probability that he or she will succeed is 0.9.

3. The Informed Source.

Venn: I don’t think you should speak of probability here at all. If you insist, you mean that the relative frequency of lasting settlements in the Middle East, in two-year periods like the upcoming one, is very low.

Pepper: There is no tendency, in this situation, for a lasting settlement in the next two years.

De Finetti: My personal probability for a lasting settlement over the next two-year period is zilch.

Keynes: Relative to the available evidence, it is not reasonable to have a high degree of belief that there will be a lasting peace settlement arising in the next two years.


Venn: The relative frequency of -success is 75%, in a reference class of students among whom the relative frequency of success is only 0.5, and who then use the kit.

Pepper: If a student has a propensity for success of only 50%, then, if she uses the kit, her propensity for success climbs to 75%.

De Finetti: The advertiser is saying something like this: If my personal betting rate on a student’s succouring is even, 1:1, then when she uses the kit, my personal betting rate changes to 3:1 on her success.

Keynes: I start with some information about the student that makes the logical probability of her success 1/3. Relative to the further information that she uses the kit, the logical probability of her success is 1/3.

You should be able to continue with Exercises 5-8 in much the same vein.

CHAPTER 13. PERSONAL PROBABILITIES

1. Nuclear power.

(a) No. The bet cannot be settled for more than half a century.

(b) Yes, if you don’t mind waiting for a couple of years.

2. Chaotic. No, because the value of the приз—to Alice—is affected by the outcome of the gamble.

3. Intelligent aliens.

(a) Yes, if you think of one year as soon enough. The condition for settling the bet are entirely definite, and the bet will be settled in one year precisely.

(b) Skill is fixed with two options. He can give $49 to Ladbrokes. He thinks he is virtually certain to collect $1 in a year. Or he can put $49 in a one-year deposit account at the bank, which at present pays a measly 4% (say). At the end of a year he collects almost $2 in interest. So he would rather put his money in the bank than make a bet that he is guaranteed to win.


5. Rating the ante: ~$25.

6. 1st bet: 1/3.

7. Make-up tests. Once again, it is your choice of numbers. The only requirement is that:

\[
\Pr(B) = \Pr(M) = 0.3
\]

Then a payoff matrix for the conditional bet would be:

<table>
<thead>
<tr>
<th>Payoff for bet against given M</th>
<th>Payoff for bet against given M</th>
</tr>
</thead>
<tbody>
<tr>
<td>Payoff for bet against given M</td>
<td>Payoff for bet against given M</td>
</tr>
<tr>
<td>$1M</td>
<td>$7</td>
</tr>
<tr>
<td>$1M</td>
<td>$7</td>
</tr>
<tr>
<td>$M</td>
<td>0</td>
</tr>
</tbody>
</table>

8. "Not" refers to preference. $10 if (2) does not occur. This seems to be a matter of logic: a preference for "$10 if (1) occurs" is the opposite of a preference for "$10 if (2) does not occur."

CHAPTER 14. COHERENCE

1. Diogenes. You bet $1 against B. He bets $9 on B. You also bet $6 on B. He bets $2 against B.

If the Leaks come in last, his net payoff is $1 - $2 = $-1.

If the Leaks do not come in last, his net payoff is $9 - $6 = $3.

2. Epicurus. You bet $7 on T. He bets $3 against T (using his first betting rate of 7).

You bet $2 against T. He bets $8 on T (using his second betting rate).

If T occurs, his net payoff is $2 - $8 = -$6.

If T does not occur, his net payoff is $7 - $8 = -$1.

3. Optimistic Cinderella. For coherence, we require that the conditional betting rate equals the betting rate on Pk6, divided by the rate on S. The conditional betting rate is 1/3. But (0.2/1/3) = 0.6, so the conditional rate is too small. (It should be 0.6, not 0.5.)

This is the situation in the payoff matrix on the other side. So Cinderella is asked to bet:

- Bet $2 on Pk6B (to win $0).
- Bet $4 against S (to win $2).
- Bet $5 against T (to win $5).

If Pk6 occurs, she wins $8, but loses her other two bets for a net loss of $1.

If (~Pk6) occurs, she loses $2 on Pk6, loses $4 against S, and wins $5 on the conditional bet, for a net loss of $1.

If S does not occur, she loses $2, wins $2, for a net loss of $1.

How did we get these numbers? In terms of p, q, r on the other side,

\[
p = 1/3, q = 0.2, \text{and } r = 1/3.
\]

The net loss should be $-p - pr = $1/2 - $1/3 = $1/30.

So we should multiply by 30 to get a net loss of $1.